

Large-scale confinement and small-scale clustering of floating particles in stratified turbulence

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We study the motion of small inertial particles in stratified turbulence. We derive a simplified model, valid within the Boussinesq approximation, for the dynamics of small particles in presence of a mean linear density profile. By means of extensive direct numerical simulations, we investigate the statistical distribution of particles as a function of the two dimensionless parameters of the problem. We find that vertical confinement of particles is mainly ruled by the degree of stratification, with a weak dependency on the particle properties. Conversely, small scale fractal clustering, typical of inertial particles in turbulence, depends on the particle relaxation time and is almost independent on the flow stratification. The implications of our findings for the formation of thin phytoplankton layers are discussed.

Particles of density different from the surrounding fluid do not follow the motion of fluid particles, and generate inhomogeneous distributions even in incompressible flows [1]. This phenomenon is crucial in a variety of instances, from cloud formation in the atmosphere, to the dynamics of plankton in the ocean and lakes, to industrial applications e.g. in reactors [2]. Inhomogeneous distribution in turbulent flows is also of interest from a theoretical point of view. In recent years, analytical, numerical and experimental studies led to significant advances in the understanding of this process [3–8]. In an incompressible turbulent flow, non-inertial, fluid particles follow the flow streamlines and remain by definition homogeneously distributed. In contrast, inertial particles with density different from the fluid, are known to accumulate in regions of high vorticity (light particles) or high strain (heavy particles) [3, 5, 9], as a consequence of the accelerations induced by the flow. Recent analytical and numerical works have shown that gravity interacts with turbulent accelerations to increase clustering of inertial particles. Moreover, turbulence can increase the settling velocity with respect to still fluid, by pushing particles in regions of downward flow [10, 11]. In presence of density fluctuations, gravity also affects the flow itself as in the case of stratified turbulence which finds many applications in natural and technological flows [12, 13]. One important example is ocean dynamics which is strongly affected by the presence of the pycnocline resulting from temperature and salinity variations [14, 15].

Remarkably, very little is known about the distribution of inertial particles in stratified turbulence, in spite of its relevance for oceanic and other applications. Recent works have studied the effect of stratification on the clustering of heavy [16] and light [17] particles and the effect of a vertical confinement in homogeneous turbulence [18]. In this Letter we investigate, by means of direct numerical simulations, the distribution of small buoyant particles transported in a turbulent stratified flow. We study both the large scale vertical distribution and the

small scale clustering of particles as a function of the relevant parameters. Unexpectedly, we find that small scale (fractal) clustering is determined by the particles relaxation time and is almost independent on the degree of stratification. Conversely, the vertical confinement of particles is mostly controlled by the degree of fluid stratification, and weakly depends on the particle properties. The dependence of the confinement on Froude number is interpreted within the framework of stratified turbulence phenomenology.

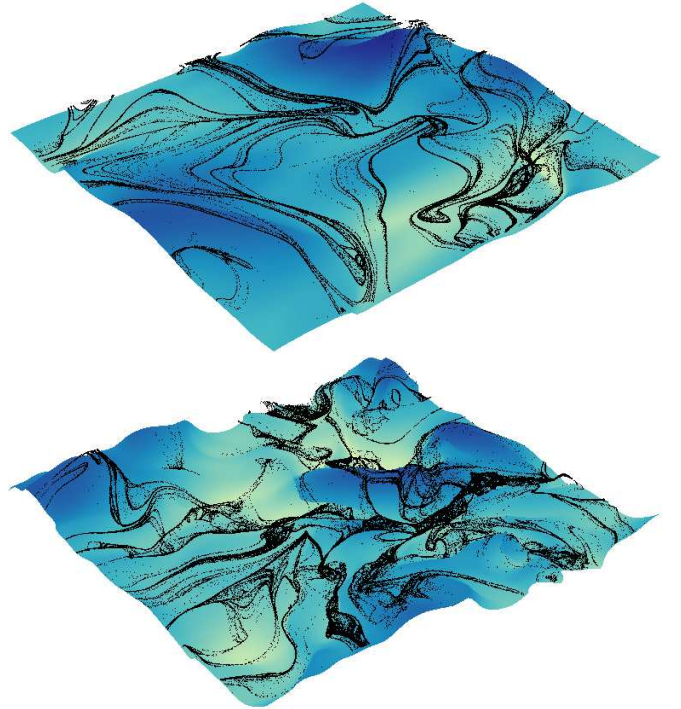


FIG. 1. Isopycnal surfaces $\theta = z$ corresponding to the particle density ρ_0 for two runs at $Fr = 0.2$ (upper plot) and $Fr = 0.3$ (lower plot). The points represent the (fractal) distribution of particles with $\tau = 0.01$ in the stationary state.

We consider a cubic box of size L of fluid linearly

(and stably) stratified in the direction z of gravity $\mathbf{g} = (0, 0, -g)$ with a constant mean density gradient $d\rho/dz = -\gamma$. Within the Boussinesq approximation, the motion for the incompressible velocity field $\mathbf{u}(\mathbf{x}, t)$ is ruled by

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\frac{1}{\rho_0} \nabla p + \nu \nabla^2 \mathbf{u} - N^2 \theta \hat{\mathbf{z}} + \mathbf{f} \quad (1)$$

$$\frac{\partial \theta}{\partial t} + \mathbf{u} \cdot \nabla \theta = \mathbf{u} \cdot \hat{\mathbf{z}} + \kappa \nabla^2 \theta \quad (2)$$

together with $\nabla \cdot \mathbf{u} = 0$. The scalar field $\theta(\mathbf{x}, t)$, which in the above equations has the dimension of a length, represents the deviations of the local density from the linear vertical profile, $\rho = \rho_0 - \gamma(z - \theta)$. ν is the kinematic viscosity, κ the density diffusivity and $N = (\gamma g / \rho_0)^{1/2}$ is the Brunt-Väisälä frequency. \mathbf{f} represents an external mechanical forcing needed to sustain turbulence. In the inviscid, unforced limit ($\nu = \kappa = \mathbf{f} = 0$) equations (1-2) conserve the total energy, sum of kinetic and potential contributions, $E = \frac{1}{2} \langle |\mathbf{u}|^2 \rangle + \frac{1}{2} N^2 \langle \theta^2 \rangle$ where $\langle \dots \rangle$ denotes the average over the domain.

The velocity \mathbf{v} of a small inertial particle transported by the flow \mathbf{u} generated by (1) evolves according to [19]

$$\frac{d\mathbf{v}}{dt} = \beta \frac{d\mathbf{u}}{dt} - \frac{\mathbf{v} - \mathbf{u}}{\tau_p} + (1 - \beta) \mathbf{g} \quad (3)$$

where $\beta = 3\rho/(\rho + 2\rho_p)$ is the density ratio (ρ_p is the density of the particle of radius a) and $\tau_p = a^2/(3\nu\beta)$ is the viscous relaxation time. We consider the limit of small particles (i.e. with small τ_p for which one can approximate $\frac{d\mathbf{u}}{dt} \simeq \frac{d\mathbf{v}}{dt}$ [19]) of density $\rho_p = \rho_0$ so that we can rewrite (3) as

$$\mathbf{v} = \mathbf{u} - \tau_p(1 - \beta) \frac{d\mathbf{v}}{dt} + \tau_p(1 - \beta) \mathbf{g} \quad (4)$$

Consistently with the Boussinesq approximation, in the limit of small τ_p , we can neglect the second term in the r.h.s. of (4) and, since $(1 - \beta) \mathbf{g} \simeq -\frac{2}{3} N^2 (z - \theta) \hat{\mathbf{z}}$, we obtain a simplified expression for the velocity of the floater whose position \mathbf{x} evolves according to

$$\frac{d\mathbf{x}}{dt} = \mathbf{v} = \mathbf{u} - \frac{1}{\tau} (z - \theta) \hat{\mathbf{z}} \quad (5)$$

where $\tau \equiv 3/(2N^2\tau_p)$ represents the relaxation time of the particle onto the isopycnal surface of density $\rho = \rho_0$. This surface, defined implicitly by the relation $z = \theta(x, y, z)$, will be denoted as $h(x, y)$, keeping in mind that in general it can be multivalued. We remark that more general models for the motion of floaters in stratified turbulence are possible, at the price of increased complexity and number of parameters [17].

Although the fluid velocity \mathbf{u} is incompressible, the velocity field transporting the floaters is not since $\nabla \cdot \mathbf{v} = -(1 - \partial\theta/\partial z)/\tau$ which is in general nonzero. Formally,

N	ν, κ	k_f	ε_I	η	τ_η	Re	Fr	τ
128	5×10^{-3}	1.0	0.195	0.028	0.16	430	0.2 – 1.0	0.01 – 10.0
256	4×10^{-3}	1.0	0.195	0.024	0.14	540	0.2 – 1.0	0.01 – 10.0

TABLE I. Parameters of the simulations. N resolution, ν and κ kinematic viscosity and diffusivity, k_f forcing wavenumber, ε_I energy input rate, $\eta = (\nu^3/\varepsilon_I)^{1/4}$ Kolmogorov scale, $\tau_\eta = (\nu/\varepsilon_I)^{1/2}$ Kolmogorov timescale, $Re = U_{rms}L/\nu$ and $Fr = \varepsilon^{1/3}k_f^{2/3}/N$. Re is computed at $Fr = 1$ at which the flow is almost isotropic, as U_{rms} varies with Fr . The forcing scale is defined as $L = \pi/k_f$. All the simulations are performed at Schmidt number $Sc = \nu/\kappa = 1$.

this expression represents the rate of contraction of the phase space (here the configuration space) under the dynamics. When it is negative we expect that trajectories of floaters will collapse on a (dynamical) fractal attractor in the phase space. An example of the attractor is displayed in Fig. which shows that the *large scale* confinement in the vertical direction coexists with a *small scale* clustering with fractal distribution on the isopycnal surface.

We have integrated the Boussinesq equations (1-2) in a domain of size $L = 2\pi$ with periodic boundary conditions by means of a fully parallel pseudo-spectral code at resolution up to $N = 256$. Turbulence is generated by a δ -correlated in time isotropic forcing \mathbf{f} which is active on a spherical shell of wavenumber around $k_f = 1$ and which pumps energy at the fixed rate ε . These parameters define, together with the Brunt-Väisälä frequency, the Froude number $Fr \equiv (\varepsilon^{1/3}k_f^{2/3})/N$ which measures the (inverse) stratification. We remark that, in stratified turbulence, the mean kinetic energy dissipation at small scales ε_ν is typically smaller than the input ε and depends on Fr , because a fraction ε_κ of the energy input is converted into potential energy during the turbulent cascade and dissipated by diffusivity [20]. Another relevant parameter in stratified turbulence is the buoyancy Reynolds number $Re_b = \varepsilon/(\nu N^2)$ defined in terms of the ratio of the buoyancy (Ozmidov) scale $\ell_B = \varepsilon^{1/2}/N^{3/2}$ to the dissipative scale $\ell_D = \varepsilon^{3/4}/\nu^{1/4}$ as $Re_b = (\ell_B/\ell_D)^{4/3}$, in analogy to the usual Reynolds number $Re = (L/\ell_D)^{4/3}$. These three numbers are not independent since $Re_b = Fr^2 Re$ [21] and $Re_b = 1$ discriminates between stratified-viscous flow ($Re_b < 1$) and stratified turbulence ($Re_b > 1$) [21]. Our simulations are within the turbulent regime, as Re_b is in the range $17 \leq Re_b \leq 540$. Together with the (1-2), we integrated the equation (5) for the particle motion for a set of 10 classes of particles characterized by different values of τ in the range $0.01 \leq \tau \leq 10.0$. In presenting the results, this time will be made dimensionless with the Kolmogorov time $\tau_\eta = (\nu/\varepsilon)^{1/2}$ by introducing a "Stokes number" $St \equiv \tau/\tau_\eta$. Table I reports the most important parameters of the simulations.

In Figure 2 we show vertical sections (at $y = 0$) of the isopycnal surface h (obtained from the solution of $z = \theta(x, y, z)$) together with positions of the particles on the same sections, for different values of the parameters Fr and τ . It is evident that the isopycnal surface h is almost flat for strong stratification and it becomes more bent (and multivalued) as Fr increases. Figure 2 shows also the effect of the relaxation time τ on the particles. When $\tau/\tau_\eta < 1$ the particles are practically attached to the isopycnal surface, while their positions depart from the surface h by increasing τ .

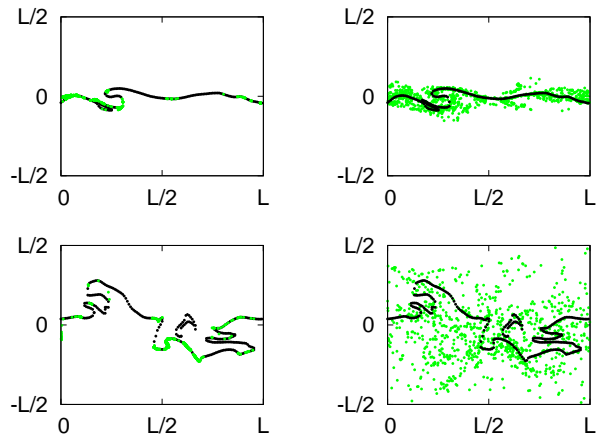


FIG. 2. (Color online) Vertical (x, z) sections of isopycnal surfaces h (black points) for $Fr = 0.3$ (upper row) and $Fr = 1.0$ (lower row) together with the positions of particles (green points) with $St = 0.6$ (left column) and $St = 60.0$ (right column).

The probability density functions (PDF) of the vertical displacement of particles with respect to their equilibrium position $z = 0$ in the absence of turbulence is shown in Fig. 3 for different values of Re , Fr and τ . We found that in the wide range of parameters investigated these distributions are close to Gaussian (with some possible deviations in the tails).

The standard deviations σ_z of these vertical distribution of particles for different values of Fr and St are shown in Fig. 4. We obtain a linear scaling of σ_z on Fr , with a coefficient which shows a (weak) dependency on St . For stronger stratification, $Fr \leq 0.3$, the standard deviation is almost independent on St , a feature which can be understood by looking at the plots in Fig. 2. It is indeed evident that for St not too large, the effect of the relaxation term in (5) is to allow the particle to detach from the level $z = \theta$ and to remain “suspended” for a time of order τ before feeling the vertical velocity towards the isopycnal surface. Because in stratified turbulence vertical velocity and vertical gradient of vertical velocity are suppressed [21], there is no mechanism which ejects particles far from the surface. Therefore, as shown

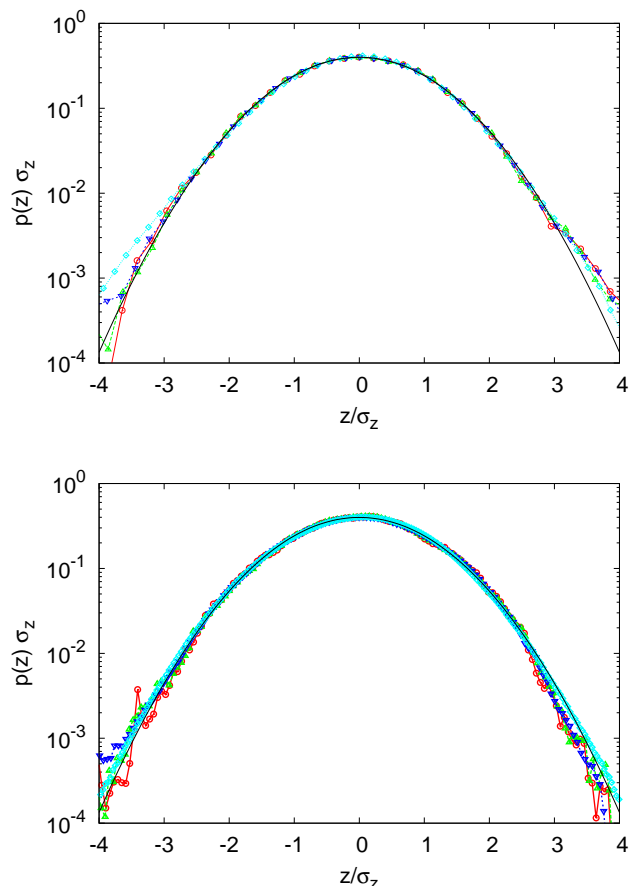


FIG. 3. Probability density functions of vertical positions of tracers for $Re = 430$, $Fr = 0.3$ (upper plot) and $Re = 540$, $Fr = 1.0$ (lower plot). In both cases the PDF for different classes of floaters, corresponding to $St = 0.06$, 0.6 , 6.0 and 60.0 are shown.

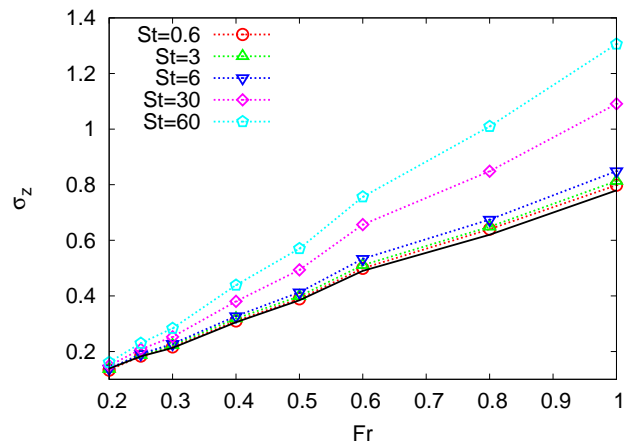


FIG. 4. Standard deviations σ_z of the PDFs of particle vertical position as a function of Fr for different values of St at $Re = 430$. The solid black line represents the standard deviation σ_h of the isopycnal surface.

by Fig. 2, the vertical region visited by particles reflects the vertical extension of the isopycnal surface and, therefore, is independent on τ . By increasing Fr , the vertical components of the velocity and of the velocity gradient increase and this produces a displacement of particles from the isopycnal surface when the relaxation time τ is sufficiently large.

In the limit of small St , the standard deviations of particles σ_z collapse on the standard deviation σ_h of the isopycnal surface. The linear dependence on Fr shown in Fig. 4 can be understood within the framework of stratified turbulence, as a manifestation of the presence of the so-called *vertical shear layers* [22] and the associated vertical correlation scale of velocity L_v . Physically this scale represents the vertical displacement for converting injected kinetic energy into potential energy and can be estimated simply as $L_v \simeq U/N$ (U is a typical large scale velocity) and therefore one obtains $L_v \propto Fr$, i.e. linear scaling as shown in Fig. 4.

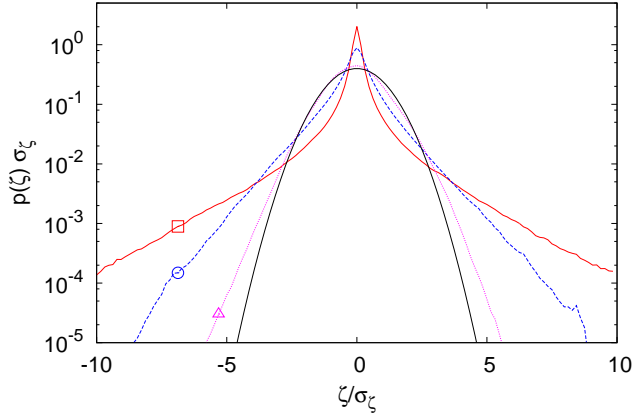


FIG. 5. Probability density functions of the variable $\zeta = z - \theta$ for $Re = 540$, $Fr = 1.0$ and $St = 1.4$ (red line with square), $St = 7.0$ (blue line with circle) and $St = 70$ (pink line with triangle). The black line is a Gaussian.

The deviation of the particles from the isopycnal surface can be investigated by looking at the statistics of the variable $\zeta = z - \theta$. The evolution of this quantity is obtained by the time evolution of the field θ along the trajectory of a floater moving with the velocity \mathbf{v} and which gives (neglecting the diffusive term) $d(z - \theta)/dt = -(z - \theta)(1 - \partial\theta/\partial z)/\tau$. In the absence of fluctuations ($\theta = 0$) this equation would simply represent the linear relaxation of particles towards the isopycnal layer $z = 0$ [18]. This is not achieved since the term $\partial\theta/\partial z$ is fluctuating without a definite sign. Figure 5 shows the normalized PDF of the variable ζ for three different values of τ at $Fr = 1.0$. It is evident that the statistics is neither Gaussian nor scale invariant and the PDF develops large tails for small relaxation times. These large fluctuations are due to the folding of

the isopycnal surface. Particles located in the neighborhood of a fold in which the isosurface is almost vertical (see examples in Figure 2) are not restored horizontally to the close-by branch they just left, but rather displaced vertically by buoyancy toward the nearest branch above or below them. This mechanism, which is enhanced at large Fr , produces a sudden increase in the distance between the particles and the isopycnal surface, and causes the development of large tails in the PDF of ζ .

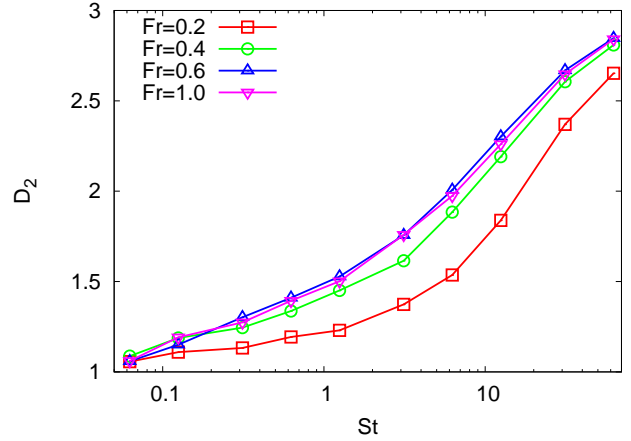


FIG. 6. Correlation dimension D_2 versus relaxation time St for different values of stratification parameter Fr .

As already discussed, floating particles moving according to (5) are transported by a compressible velocity field and are therefore expected to relax on a (dynamical) fractal subset of the physical space, as shown in the examples of Fig. . In order to characterize this subset, and its dependence on the parameters, we have measured the correlation dimension D_2 of particle distribution, defined as the scaling exponent of the probability of finding two particles at distance less than r : $P(|\mathbf{x}_1 - \mathbf{x}_2| < r) \propto r^{D_2}$ as $r \rightarrow 0$ [23]. The maximum value $D_2 = 3$ denotes uniformly distributed particles, while $D_2 < 3$ indicates fractal patchiness with smaller D_2 corresponding to more clustered distributions and increased probability of finding pairs of particles at close separation. Figure 6 shows D_2 as a function of St for different values of Fr . In the limit $St \gg 1$, for which $\mathbf{v} \rightarrow \mathbf{u}$ in (5), floaters move as fluid particles in an incompressible velocity and therefore remain uniformly distributed in the volume with $D_2 = 3$. The fractal dimension is found to be monotonic in St and attains a minimum value $D_2 \simeq 1$ for the smallest relaxation time. This value indicates distributions of particles on quasi-one-dimensional structures (as shown qualitatively in Fig.) which is almost independent on Fr . This is a remarkable result, as one could expect that for strong stratification and small τ , in which particles lie on the isopycnal surface which is almost flat, the fractal dimension would be close to $D_2 = 2$. The fact that we find $D_2 \simeq 1$ indicates that the dynamics on the surface is

dissipative and therefore the horizontal velocity field is compressible.

The weak dependence of the correlation dimension on the stratification extends also to larger values of St and we find that, in general, D_2 is virtually independent on Fr for $Fr \geq 0.5$. This is in contrast with the (large scale) vertical confinement shown in Fig. 5 which displays a strong (linear) dependence on Fr and a weak dependence on St .

In conclusion, we have investigated the dynamics of neutrally buoyant particles in a turbulent stratified flow. We have shown that the extension σ_z of the vertical layer in which the particles are confined depends on the characteristics of the flow and moderately on the size of the particles. On the contrary, the small scale patchiness inside this layer, expressed by the correlation dimension, strongly depends on the particle size and only weakly on the stratification of the flow.

One of the most remarkable examples of confinement of particles in the ocean is the formation of the so-called thin phytoplankton layers (TPL): aggregations of phytoplankton and zooplankton at high concentration with thickness from centimeters to few meters, extending up to several kilometers horizontally and with timescale from hours to days [24].

Among the different mechanisms proposed for the formation of TPLs, buoyancy force in stratified flow is particularly relevant for non-swimming species and aggregates, such as diatoms and marine snow, which are often observed to accumulate in correspondence of strong stratification [25]. In order to test the applicability of our model in the aquatic ecosystems we provide a simple example using the typical values observed for diatom-dominated marine snow [25]. Assuming an energy dissipation rate $\varepsilon \sim 10^{-8} \text{ m}^2 \text{ s}^{-3}$ ($\tau_\eta \simeq 10 \text{ s}$) and a Brunt-Väisälä frequency $N \sim 0.1 \text{ s}^{-1}$, aggregates of size $a \simeq 0.5 \text{ cm}$ have a relaxation time $\tau \simeq 18 \text{ s}$ which corresponds to $St = 1.8$, inside the range in which we observe clustering at small scales (Fig. 6) for all the values of Fr . Smaller aggregates or single cells of size $a = 0.1 \text{ cm}$ correspond to $St \simeq 45$ and, according to our results, would distribute almost homogeneously (with a fractal dimension close to 3) within the thin layer.

Beside the applications to thin layers, our results are of general interest as they describe quantitatively and for the first time how the combination of turbulence and

stratification generates both large scale confinement and small scale fractal patchiness in a suspension of buoyant particles.

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